Engineering Notes

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Use of Flow Variable Gradients in Design Calculations of Inlets

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Nomenclature

K_S = curvature in meridional plane of reference line
M = Mach number
n = arc length normal to streamline
p = pressure
q = velocity
r = radial coordinate
s = arc length along streamline
β = angle of reference line measured from the velocity direction in front of the reference line
γ = ratio of specific heats
ϵ = 0 or 1 for 2D or axisymmetric flow, respectively
θ = flow angle measured from the velocity direction in front of reference line
ρ = density

ONE purpose of this Note is to point out some uses of flow variable gradients in the calculation of flowfields necessary in the design of inlets. These gradients can be obtained directly and algebraically from the equations of motion; that is, there is no need to solve differential equations. There are two ways in which this information can be used. It can provide a check on the accuracy of a numerical solution, e.g., by a method of characteristics program, or it can provide more accurate input for such a program. To illustrate the latter point, rather than use only the initial shock angle at the sharp lip of the cowl, the curvature of the shock can be specified. The first of these, a check on accuracy, is perhaps the more practical way to use the information in conjunction with an existing computer program.

Although there are several points in the inlet flowfield where such local information on the gradients is easily obtainable, the specific case of the leading edge of a cowl with a sharp lip will be used for illustrative purposes. This leads to the second purpose of this note, viz., to bring to the attention of those using or developing programs the existence of tables[1] which make quite convenient the determination of pressure or velocity gradients and shock curvature. These tables do have a limitation, since uniform flow in front of the shock is assumed in that derivation. They would be directly applicable to computing the flow over open-nosed bodies of revolution (see, e.g., the computations reported in Ref. 2) or to inlets with a centerbody if the bow shock intersects the cowl downstream of the leading edge.

It is well known, and illustrated in Ref. 1, that the flow variable gradients (velocity, pressure, etc.) are linearly related to the two curvatures of a reference line (which will be taken later as a shock). The two curvatures are K_S in the meridional plane and $1/r$ in the azimuthal plane for axisymmetric flow; for two-dimensional flow the coefficient of the $1/r$ term is set to zero. The derivation of these linear relations is straight-forward if the equations of motion are written in terms of natural coordinates s and n; thus, there is no need to repeat the steps here. However, it is worth noting that the derivation is simplier if p and θ are used as dependent variables rather than q and θ as in Ref. 1; this is because with p and θ, entropy does not appear explicitly.

Using the notation shown in Fig. 1 and using q and θ, the linear relations mentioned previously, as given in Ref. 1, are

$$\partial\theta/\partial s = F_1 K_S + F_2(\epsilon/r) \tag{1}$$

$$(1/q_1)\partial q/\partial s = F_3 K_S + F_4(\epsilon/r) \tag{2}$$

where $F_i = F_i(\gamma, M_1, \beta)$, the subscript 1 refers to conditions upstream of the reference line with slope $\tan\beta$, and $\epsilon = 0$ or 1 for two-dimensional or axisymmetric flow, respectively. These relations require an inviscid flow in chemical equilibrium with constant γ. In Ref. 1, the F_i are tabulated for $\gamma = 1.4$ and $1.1 \leq M_1 \leq 10$ as functions of β, with the reference line taken as a shock wave and uniform flow upstream of the shock. (Note that in Ref. 1 three additional functions are tabulated; these are used to calculate the initial slope of constant density and constant Mach number contour lines behind the shock. See Ref. 3 for a discussion of these slopes.)

An example of the use of these relations is as follows. If the leading edge of the cowl is sharp and supports an attached shock, the curvature of the shock is obtained from Eq. (1) since the curvature of the cowl, $\partial\theta/\partial s$, and r are known. With this value of K_S, Eq. (2) gives $\partial q/\partial s$, and then the pressure gradient is obtained from the momentum equation

$$\partial p/\partial s + \rho q(\partial q/\partial s) = 0 \tag{3}$$

Of course there are many other ways of using Eqs. (1) and (2) to obtain useful local flowfield information in either a direct or inverse manner.

With the restriction to uniform flow in front of the shock, the tables of Ref. 1 cannot be used in the more common situation in which the bow shock is upstream of the cowl lip. There are several method of characteristic programs described in the literature for the direct problem,[4,5] or partially inverse problem.[6] In order to use the gradients for an accuracy check of results from such programs, it is useful to record the form these take without the uniform flow restriction. Using Eq. (3) and the continuity equation with gradients of ρ and q eliminated, one obtains

$$\partial\theta/\partial s = -f_1 K_S - f_2(\epsilon/r) \tag{4}$$

$$\partial p/\partial s = -f_3 K_S - f_4(\epsilon/r) \tag{5}$$

where the f_i are functions of γ, M_1, β, and $\partial M_1/\partial\beta$. Specifically,

$$f_1 = [\rho q^2 \cos(\beta - \theta)\partial\theta/\partial\beta + (M^2 - 1) \times \sin(\beta - \theta)\partial p/\partial\beta]/\rho q^2 D \sin^2(\beta - \theta)$$

$$f_2 = \sin\theta \cot(\beta - \theta)$$

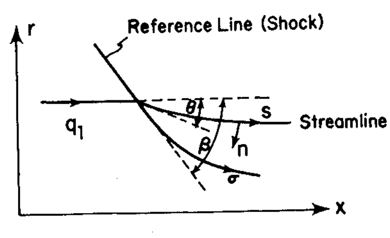

Fig. 1 Coordinates and notation for derivation of gradients.

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$$f_3 = [\rho q^2 \sin(\beta - \theta)\partial\theta/\partial\beta + \cos(\beta - \theta)\partial p/\partial\beta]/D \times \sin^2(\beta - \theta)$$

$$f_4 = D^{-1}\rho q^2 \sin\theta \qquad D = M^2 - 1 - \cot^2(\beta - \theta)$$

where $\partial\theta/\partial\beta$ and $\partial p/\partial\beta$ can be written in terms of $\partial M_1/\partial\beta$ through the shock relations if the reference line is a shock.

The reference line need not be a shock. In the approach described in Ref. 6, the cowl surface is calculated rather than specified. In that case the reference line (initial data line in Ref. 6) might be a characteristic. Then Eqs. (4) and (5) could still be used with $\partial\theta/\partial\beta$ and $\partial p/\partial\beta$ calculated from the compatibility relations.

Although the tabulations of Ref. 1 are for $\gamma = 1.4$, the form of the gradients given in Eqs. (4) and (5) are not restricted to a constant γ gas. They apply to a flow with chemical reactions provided that chemically the flow is either frozen or in equilibrium. If the reference line is a shock, the appropriate shock relations are used to calculate the f_i. For nonequilibrium flow additional terms appear on the right-handside of Eqs. (4) and (5); these terms are proportional to the "production" terms in the rate equation. See Ref. 7 for the result with vibrational nonequilibrium included or Ref. 8 for a discussion of the general case with an arbitrary number of chemical reactions. The latter reference also discusses the use of other checks and the need for these in characteristic calculations.

In the previous discussion it was assumed that $r \neq 0$ since, for axisymmetric flow, special consideration of the axis is necessary. A kind of branch point singularity occurs there. The gradients and shock curvature can still be calculated for a given streamline (centerbody) curvature but ordinary differential equations must be solved. It is then a matter of individual judgement whether the effort to obtain this information is justified. The procedure, some results, and further references for getting this gradient information are given in Ref. 9.

Finally, gradients and shock curvature could be obtained at a point of shock reflection in the inlet (boundary-layer effects excluded). They would be obtained from Eqs. (4) and (5) using the nonuniform flow behind the incident shock to calculate the curvature of the reflected shock and surface gradients behind it.

References

[1] Gerber, N. and Bartos, J. M., "Tables for Determination of Flow Variable Gradients Behind Curved Shock Waves," Rept. R-1086, Jan. 1960, Ballistics Research Lab., Aberdeen Proving Ground, Md.; for summary, see *Journal of Aerospace Sciences*, Vol. 27, No. 12, Dec. 1960, pp. 958–959.

[2] Presley, L. L. and Mossman, E. A., "A Study of Several Theoretical Methods for Computing the Zero-Lift Wave Drag of a Family of Open-Nosed Bodies of Revolution in the Mach Number Range of 2.0 to 4.0," TN 4368, Sept. 1958, NACA.

[3] Hayes, W. D. and Probstein, R. F., *Hypersonic Flow Theory*, 2nd ed., Vol. 1, Academic Press, New York, 1966, p. 396.

[4] Sorensen, V. L., "Computer Program for Calculating Flow Fields in Supersonic Inlets," TN D-2897, July 1965, NASA.

[5] Sorensen, N. E., Latham, E. A., and Morris, S. J., *Prediction of Supersonic and Hypersonic Inlet Flow Field*, SP-228, NASA, 1970.

[6] Andersen, B. H., *Optimization of Supersonic Inlets Using the Method of Characteristics*, SP-228, NASA, 1970.

[7] Sedney, R., "Some Aspects of Nonequilibrium Flows," *Journal of Aerospace Sciences*, Vol. 28, No. 3, March 1961, pp. 189–196, 208.

[8] Sedney, R., "Method of Characteristics," *Gas Dynamics: A Series of Monographs, Vol.1, Pt. II, Nonequilibrium Flows*, edited by P. P. Wegener, Marcel Decker, New York, 1970, Chap. 4, pp. 159–225.

[9] Sedney, R. and Gerber, N., "Shock Curvature and Gradients at the Tip of Pointed Axisymmetric Bodies in Nonequilibrium Flow," *Journal of Fluid Mechanics*, Vol. 29, Pt. 4, Sept. 1967, pp. 765–779.

Energy State Approximation and Minimum-Fuel Fixed-Range Trajectories

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Introduction

THE energy state approximation[1] would appear to be a powerful tool for performance optimization. However, a dilemma appears in its application to minimum-fuel problems. Namely, for aircraft like the F4, the application of the Maximum Principle[2] fails to yield any solutions to the problem of a maximum range cruise. This will be shown to be a consequence of the fact that the velocity set[2] is not convex, allowing relaxed controllers that attain fuel economies superior to any control satisfying the Maximum Principle.

When an optimal control does fail to exist, there are suboptimal trajectories that achieve fuel economies superior to the full-powered climbs and zero throttle glides described in Ref. 1. These suboptimal trajectories contain a minimum fuel cruise segment, and achieve fuel economies very close to those of the optimum relaxed controller. Generally, it has been found that optimal controllers cannot exist when the aerodynamic data includes a classical minimum-fuel cruise point[3] within the flight envelope. They may exist otherwise, and then will correspond to that described in Ref. 1.

Finally, the climb path to the suboptimal cruise point may be well approximated by a Rutowski[4] minimum-fuel energy-climb path. This suggests that the minimum-fuel problem with range constraints can be well treated by combining minimum-fuel energy-climbs with classical cruises and maximum-range glides.

Energy State Equations

The energy state approximation assumes that altitude and velocity may be interchanged through zero-cost, constant-energy zooms and dives. The energy per unit weight, specific energy, is given by

$$E = V^2/2g + h \tag{1}$$

with V and h the velocity and altitude. The fuel rate $\dot{m}$ is assumed given by

$$\dot{m} = -\sigma T \tag{2}$$

where σ is the thrust specific fuel consumption and T the thrust. The energy state equations, with range x as the independent variable, are,[1]

$$dE/dx = [T(E,V,\pi) - D(E,V)]/W \tag{3}$$

$$dm/dx = -\sigma(E,V,\pi)T(E,V,\pi)/V \tag{4}$$

where π is the throttle angle, W is the (assumed constant) weight, and D is drag.

Drag, thrust and specific fuel consumption are indicated as functions of E and V rather than h and V to allow later use of V as a control variable. The limits on V are

$$Vs \leq V \leq (2gE)^{1/2} \tag{5}$$

where Vs is the stall velocity.

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